Control of spin dynamics in a two-dimensional electron gas by electromagnetic dressing

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We solved the Schrödinger problem for a two-dimensional electron gas (2DEG) with the Rashba spin-orbit interaction in the presence of a strong high-frequency electromagnetic field (dressing field). The found eigenfunctions and eigenenergies of the problem are used to describe the spin dynamics of the dressed 2DEG within the formalism of the density matrix response function. Solving the equations of spin dynamics, we show that the dressing field can switch the spin relaxation in the 2DEG between the cases corresponding to the known Elliott-Yafet and D'yakonov-Perel' regimes. As a result, the spin properties of the 2DEG can be tuned by a high-frequency electromagnetic field. The present effect opens an unexplored way for controlling the spin with light and, therefore, forms the physical prerequisites for creating light-tuned spintronics devices.

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I. INTRODUCTION

One of the most exciting trends in modern condensed matter physics is using the electron spin-of-freedom to store and transfer information. This field of research — which is known as spintronics — opened a way for various high-performance devices which have a number of important advantages as compared to conventional electronics, including growth in data processing speed, reduction in power consumption, etc[1–6]. Besides successful spintronic experiments based on various ferromagnetic structures[7–9], an alternative approach to use nonmagnetic semiconductor nanostructures with spin-orbit interaction is actively investigated in recent years[10–12]. Therefore, the study of spin transport in a two-dimensional electron gas (2DEG) with the spin-orbit interaction is currently in the focus of attention. One of the most important characteristics of spintronics devices is the spin relaxation time which describes the spin evolution. Since it is responsible for the spin transfer of information, the search of ways to control this time is interesting from both fundamental and applied viewpoint. In the present paper, we report a novel method to control the spin relaxation time of 2DEG with a strong high-frequency electromagnetic field.

It is well-known that the interaction between electrons and a strong high-frequency electromagnetic field cannot be described as a weak perturbation. In this case, the system "electron + electromagnetic field" should be considered as a whole. Such a bound electron-field system, which was called "electron dressed by field" (dressed electron), became a commonly used model in modern physics[13,14]. Recently, we demonstrated that strong interaction between 2DEG and a high-frequency electromagnetic field drastically suppresses the scattering of dressed electrons[15,16]. Since the spin relaxation depends on both the mechanism of spin-orbit interaction and scattering processes, one can expect that the spin relaxation time is strongly affected to the dressing electromagnetic field. Although various mechanisms of spin evolution in 2DEG have been studied in details both theoretically and experimentally (see, e.g., Refs. 17–20), the spin dynamics of electromagnetically dressed 2DEG escaped the attention before. The present study is aimed to fill partially this gap at the border between spintronics and quantum optics.

II. THE SPIN HAMILTONIAN OF DRESSED 2DEG

For definiteness, we will restrict our consideration to a 2DEG with the Rashba spin-orbit interaction, which is subjected to a plane monochromatic linearly polarized electromagnetic wave propagating perpendicularly to the 2DEG plane (see the insert in Fig. 1). In what follows, we will assume that the wave frequency, ω_0, meets two conditions. Firstly, the wave frequency is far from resonant electron frequencies corresponding to interband electron transitions and, therefore, the interband absorption of the wave by the 2DEG is absent. Secondly, the wave frequency is high enough in order to satisfy the inequality $\omega_0\tau_0 \gg 1$, where τ_0 is the electron scattering time in an unirradiated 2DEG. It is well-known that the intraband (collisional) absorption of wave energy by conduction electrons is negligibly small under this condition (see, e.g., Refs. 21,22). Thus, the considered electromagnetic wave can be treated as a purely dressing (nonabsorbable) field. In the absence of scatterers, the wave function of a dressed electron satisfies the non-stationary Schrödinger equation with the Hamiltonian

$$\hat{\mathcal{H}} = \frac{1}{2m}\left(\hbar\mathbf{k} - e\mathbf{A}\right)^2 + \alpha\left[\boldsymbol{\sigma}\times\left(\hbar\mathbf{k} - e\mathbf{A}\right)\right]_z, \qquad (1)$$

where $\mathbf{k}=(k_x,k_y)$ is the wave vector of the electron in the 2DEG, m is the effective electron mass in the 2DEG, e is the electron charge, $\mathbf{A}=(\mathbf{E}_0/\omega_0)\cos\omega_0 t$ is the vector potential of the electromagnetic wave, $\mathbf{E}_0=(0,E_0,0)$ is the electric field amplitude of the wave which is assumed to be linearly polarized along the y axis, $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ is the Pauli matrix vector and α is the Rashba spin-orbit coupling constant. To simplify calculations, let us subject the Hamiltonian (1) to the unitary transformation

$$\hat{U} = \frac{1}{\sqrt{2}} e^{i\left(\frac{k_y e E_0}{m\omega_0^2}\sin\omega_0 t-\frac{e^2E_0^2 t}{4m\omega_0^2\hbar}-\frac{e^2E_0^2}{8m\omega_0^3\hbar}\sin 2\omega_0 t\right)} \times \begin{pmatrix} e^{i\frac{\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} & e^{-i\frac{\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} \\ e^{i\frac{\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} & -e^{-i\frac{\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} \end{pmatrix}. \quad (2)$$

Then the transformed Hamiltonian (1),

$$\hat{\mathcal{H}}' = \hat{U}^\dagger \hat{\mathcal{H}} \hat{U} - i\hbar \hat{U}^\dagger \frac{\partial U}{\partial t},$$

takes the form

$$\hat{\mathcal{H}}' = \begin{pmatrix} \hbar^2k^2/2m+\alpha\hbar k_y & -i\alpha\hbar k_x e^{-i\frac{2\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} \\ i\alpha\hbar k_x e^{i\frac{2\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t} & \hbar^2k^2/2m-\alpha\hbar k_y \end{pmatrix}. \quad (3)$$

Seeking solutions of the Schrödinger equation with the Hamiltonian (3) in the form

$$\psi = \begin{pmatrix} a_+ \\ a_- \end{pmatrix} \quad (4)$$

and substituting the spinor (4) into the non-stationary Schrödinger equation with the Hamiltonian (3), $i\hbar\partial\psi/\partial t=\hat{\mathcal{H}}'\psi$, we arrive at the system of differential equations

$$i\dot{a}_\pm = \left(\frac{\hbar k^2}{2m}\pm\alpha k_y\right)a_\pm \mp i\alpha k_x a_\mp e^{\mp i\frac{2\alpha e E_0}{\hbar\omega_0^2}\sin\omega_0 t}. \quad (5)$$

Let us apply the Jacobi-Anger expansion[23],

$$e^{iz\sin\gamma} = \sum_{n=-\infty}^{\infty} J_n(z)e^{in\gamma},$$

to the exponents in the right side of Eqs. (5) and assume the 2DEG to fill electronic states under the Fermi energy $\varepsilon_F=\hbar^2k_F^2/2m$. Then Eqs. (5) take the form which is mathematically equal to the equations of quantum dynamics of a two-level quantum system under periodical pumping, which are analyzed in details in conventional textbooks on quantum mechanics. If the photon energy $\hbar\omega_0$ is much large than both the Fermi energy ε_F and the spin-orbit interaction energy $\alpha\hbar k_F$, the high-frequency harmonics $e^{in\omega_0 t}$ with $n\neq 0$ in the Jacobi-Anger expansion ("non-resonant terms") make negligibly small contribution to solutions of the quantum dynamics equations (5) and can be omitted (see, e.g., the similar analysis for a two-level quantum system under a periodic pumping in Ref. 24). Therefore, Eqs. (5) can be rewritten for the considered high-frequency dressing field as

$$i\dot{a}_\pm = \left(\frac{\hbar k^2}{2m}\pm\alpha k_y\right)a_\pm \mp i\alpha k_x a_\mp J_0\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right), \quad (6)$$

where $J_0(x)$ is the zeros order Bessel function of the first kind. The equations (6) can be solved trivially and we arrive at the sought two wave functions (4),

$$\psi_\pm(\mathbf{k}) = \begin{pmatrix} \left[\frac{\sqrt{k_y^2+J_0^2\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right)k_x^2}\pm k_y}{2\sqrt{k_y^2+J_0^2\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right)k_x^2}}\right]^{\frac{1}{2}} \\ \pm i\left[\frac{\sqrt{k_y^2+J_0^2\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right)k_x^2}\mp k_y}{2\sqrt{k_y^2+J_0^2\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right)k_x^2}}\right]^{\frac{1}{2}} \end{pmatrix} e^{-\frac{i\varepsilon_\pm(\mathbf{k})t}{\hbar}}, \quad (7)$$

which correspond to the two spin split branches of energy spectrum of dressed 2DEG,

$$\varepsilon_\pm(\mathbf{k}) = \frac{\hbar^2k^2}{2m}\pm\alpha\hbar\sqrt{k_y^2+J_0^2\left(\frac{2\alpha e E_0}{\hbar\omega_0^2}\right)k_x^2}. \quad (8)$$

III. SPIN DYNAMICS OF DRESSED 2DEG

In order to analyze the spin dynamics of the dressed 2DEG under the influence of scattering processes, let us use a conventional formalism based on the density matrix response function[25–29]. The comprehensive reviews of this theoretical technique can be found, for instance, in Refs. 30–32. Within this approach, the evolution of the electron spin $\mathbf{S}=(S_x,S_y,S_z)$ can be described by the diffusion equation, $D^{-1}\mathbf{S}=0$, where D is the inverse propagator of the spin density fluctuation, which is also known as a diffuson. Assuming the scattering processes in 2DEG to be caused by a short-range "white noise" disorder, the diffuson can be easily calculated by applying the standard diagram technique[25,29]. Writing the diffuson as a sum of single joint scattering events diagrams I_{ij}, we arrive at the expression $D=(1-I_{ij})^{-1}$. In the case of spatially uniform electron distribution, the scattering event diagram at the Fermi level can be expressed in terms of retarded and advanced Green's functions and is given by

$$I_{ij} = \frac{\hbar}{2\pi\nu_F\tau}\sum_{\mathbf{k}_F}\mathrm{Tr}[G^A(\mathbf{k},\varepsilon_F)\sigma_i G^R(\mathbf{k},\varepsilon_F+\hbar\omega)\sigma_j], \quad (9)$$

where ν_F is the density of states of 2DEG at the Fermi level, τ is the scattering time of 2DEG at the Fermi level, and $i,j=x,y,z$. Correspondingly, $G^{R(A)}$ in Eq. (9) is the disorder-averaged single-particle retarded (advanced) Green's function,

$$G^{R(A)}(\mathbf{k},\varepsilon_F) = \sum_{n=\pm}\frac{\psi_n(\mathbf{k})\psi_n^\dagger(\mathbf{k})}{\varepsilon_F-\varepsilon_n(\mathbf{k})\pm i\hbar/2\tau}, \quad (10)$$

which is written in the representation of wave vector $\mathbf{k}$ and frequency ω. Formally, the key expressions (10) and (9) have the same form for both unirradiated 2DEG and 2DEG subjected to a dressing field. However, for the considered case of dressed 2DEG, we have to use the wave function of dressed 2DEG (7) and the energy spectrum of dressed 2DEG (8) in order to calculate the Green's function (10). We have also to take into account that the dressing field renormalize the scattering time, τ, which takes place both in Eq. (10) and Eq. (9). Generally, the scattering time is given by the expression

$$\frac{1}{\tau} = \sum_{\mathbf{k}'} w_{\mathbf{k}_F\mathbf{k}'}, \tag{11}$$

where $w_{\mathbf{k}\mathbf{k}'}$ is the electron scattering probability per unit time between electron states with wave vectors $\mathbf{k}$ and $\mathbf{k}'$. For the dressed 2DEG, the scattering probability has the form[15]

$$w_{\mathbf{k}'\mathbf{k}} = J_0^2\left(\frac{e\mathbf{E}_0(\mathbf{k}-\mathbf{k}')}{m\omega_0^2}\right) w^{(0)}_{\mathbf{k}'\mathbf{k}}, \tag{12}$$

where $w^{(0)}_{\mathbf{k}'\mathbf{k}}$ is the scattering probability for the 2DEG in the absence of the dressing field.

To simplify calculation of the spin dynamics, let us assume that the scattering disorder is weak ($\hbar/\tau\varepsilon_F \ll 1$) and the energy of spin-orbit coupling is low ($\alpha\hbar k_F/\varepsilon_F \ll 1$). Performing the integration in Eq. (9) over the Fermi level, we get matrix elements of the diffuson. As a final result, we arrive to the spin diffusion equation $\dot{S}_{x,y,z} = -(1/\tau_{x,y,z})S_{x,y,z}$, where $\tau_{x,y,z}$ is the sought spin relaxation time for various spin projections. Since the energy spectrum of dressed 2DEG (8) is anisotropic, the spin relaxation times τ_x and τ_y are different. However, for realistic parameters of the considered problem, the spin coupling to the dressing field is very weak ($2\alpha e E_0/\hbar\omega_0^2 \ll 1$). Therefore, the anisotropy of the spin relaxation time in the 2DEG plane can be neglected and we arrive at the expression $\tau_{x,y} = \tau_z/2 \equiv \tau_s$, where

$$\tau_s = \frac{1+4\zeta^2}{2\zeta^2}\tau \tag{13}$$

is the characteristic spin relaxation time in the dressed 2DEG, $\zeta = \tau/\tau_{so}$, τ is the scattering time given by Eqs. (11)–(12), and $\tau_{so} = 1/(\alpha k_F)$ is the time of spin precession at the Fermi level, which is caused by the spin-orbit interaction.

IV. DISCUSSION AND CONCLUSIONS

It follows from Eqs. (13) that the spin relaxation time, τ_s, strongly depends on the ratio of the scattering time and the spin precession time, $\zeta = \tau/\tau_{so}$. Namely, for the case of $\zeta \gg 1$, the spin relaxation time is $\tau_s \sim \tau$. On the contrary, for the case of $\zeta \ll 1$, the spin relaxation time is $\tau_s \sim \tau_{so}$. Physically, this strong dependence of the spin relaxation time (13) on the ratio $\zeta = \tau/\tau_{so}$ arises from different mechanisms of spin relaxation, which are dominant for the cases of $\zeta \gg 1$ and $\zeta \ll 1$ (see, e.g., Refs. 33–35). If the scattering time, τ, is much larger than the spin precession time, τ_{so}, the spin relaxation is defined substantially by the scattering processes (the Elliott-Yafet (EY) spin relaxation mechanism[36,37]). The EY spin relaxation alone results in $\tau_s \sim \tau$ for the case of $\zeta \gg 1$. If the scattering time, τ, is much less than the spin precession time, τ_{so}, the spin relaxation is defined substantially by the spin-orbit interaction (the D'yakonov-Perel' (DP) spin relaxation mechanism[38]). The DP spin relaxation alone results in $\tau_s \sim \tau_{so}$ for the case of $\zeta \ll 1$. As a consequence, the nonmonotonic dependence of the spin relaxation time τ_s on the ratio τ/τ_{so} appears (see Fig. 2). For an unirradiated 2DEG, the scattering time $\tau = \tau_0$ depends only on the properties of the given nanostructure and cannot be easily changed in experiments (experimentally measured values of the scattering time τ_0 in various two-dimensional systems can be found, e.g., in Ref. 39). On the contrary, in the considered case of dressed 2DEG, the scattering time τ depends on both the dressing field amplitude E_0 and the dressing field frequency ω_0 [see Eqs. (11)–(12)] and can strongly differ from the initial scattering time in unirradiated 2DEG, τ_0. Therefore, changing the parameters of dressing field, we can change the value of the scattering time τ. As a result, the attractive possibility to switch the spin relaxation process between EY and DP regimes with a high-frequency electromagnetic field appears. To clarify the results of numerical calculations of the spin relaxation

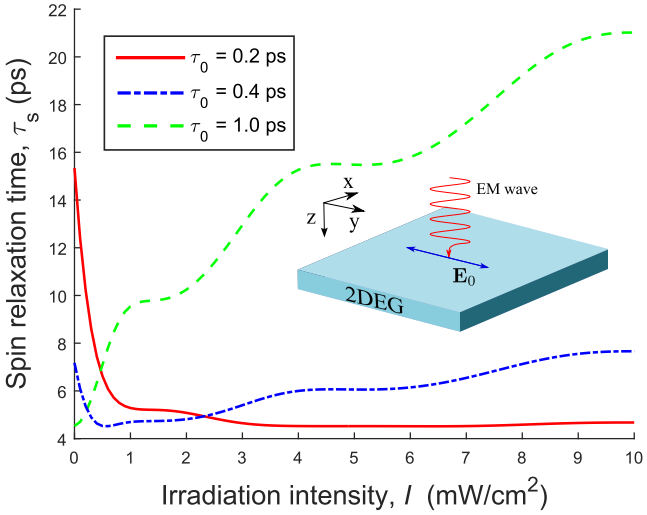

FIG. 1: (Color online) The dependence of the spin relaxation time in a 2DEG on the intensity of a dressing electromagnetic field with the frequency $\omega_0 = 100$ GHz. The 2DEG is assumed to be localized in GaAs quantum wells with different initial scattering times τ_0, the electron effective mass $m = 0.067m_0$, the Fermi energy $\varepsilon_F = 10$ meV, and the spin-orbit coupling constant $\alpha = 3.3 \times 10^3$ m/s. The insert shows the sketch of the system under consideration.

time τ_s [see Figs. 1–2], let us discuss the dependence of the scattering time (11) on the intensity of the dressing field $I = \epsilon_0 E_0^2 c/2$. It follows from the scattering probability (12) that the dependence arises from the Bessel function which decreases with increasing the intensity, I. Therefore, the scattering time in 2DEG, τ, increases with increasing intensity of the dressing field[15,16]. If the initial scattering time in unirradiated 2DEG, τ_0, is large enough (the dashed line in Fig. 1), the EY spin relaxation is dominant in the absence of the dressing field. In this case, the field-induced increase of scattering time, τ, does not change qualitatively the EY spin relaxation mechanism. As a result, the relaxation time marked by the dashed line in Fig. 1 increases monotonically with increasing the dressing field intensity. On the contrary, if the scattering time in unirradiated 2DEG, τ_0, is small enough (the solid and dot-dashed lines in Fig. 1), the DP spin relaxation is dominant in the absence of the dressing field. In this case, the field-induced increasing of scattering time, τ, switches the DP spin relaxation mechanism to the EY one. As a consequence, the relaxation times marked by the solid and dot-dashed lines in Fig. 1 demonstrate non-monotonical behavior with increasing the dressing field intensity. Therefore, the dressing field can switch the spin relaxation between DP and EY regimes in a 2DEG with strong scattering (see the insert in Fig. 2). As to the weak oscillating behavior of curves in Fig. 1, it is caused formally by the oscillating behavior of the Bessel function in the scattering probability (12).

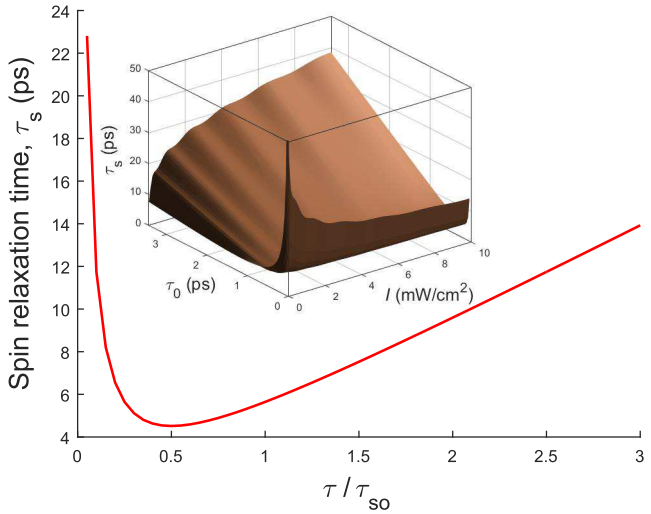

FIG. 2: (Color online) The dependence of the spin relaxation time on the ratio of the scattering time in a dressed 2DEG, τ, and the time of spin precession, τ_{so}, for the 2DEG in a GaAs quantum well with the electron effective mass $m = 0.067m_0$, the Fermi energy $\varepsilon_F = 10$ meV, and the spin-orbit coupling constant $\alpha = 3.3 \times 10^3$ m/s. The insert demonstrates the dependence of the spin relaxation time, τ_s, on the dressing field intensity, I, and the initial scattering time, τ_0, for a dressing electromagnetic field with the frequency $\omega_0 = 100$ GHz.

Summarizing the aforesaid, we can conclude that the dressing field can switch the spin relaxation mechanism in the 2DEG between the cases corresponding to the well-known Elliott-Yafet and D'yakonov-Perel' regimes. As a result, the spin properties of the 2DEG can be tuned by a high-frequency electromagnetic field. Particularly, we showed that the irradiation of 2DEG by the dressing field results in increasing the spin relaxation time. Currently, only low-frequency (particularly, stationary) magnetic and electric fields were considered as a tool to control spin properties of solids. Therefore, the present effect opens an alternative way for the spin control with light and, therefore, forms physical prerequisites for creating light-tuned spintronics devices.

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[1] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnar, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science **294**, 1488 (2001).

[2] D. D. Awschalom, D. Loss, and N. Samarth, *Semiconductor Spintronics and Quantum Computation* (Spinger-Verlag, Berlin, 2002).

[3] I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).

[4] S. D. Bader and S. S. P. Parkin, Ann. Rev. Condens. Matter Phys. **1**, 71 (2010).

[5] A. Hirohata and K. Takanashi, J. Phys. D: Appl. Phys. **47**, 193001 (2014).

[6] D. D. Awschalom, L. C. Bassett, A. S. Dzurak, E. L. Hu, J. R. Petta, Science **339**, 6124 (2013).

[7] Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D. D. Awschalom, Nature **402**, 790 (1999).

[8] X. Lou, C. Adelmann, S. A. Crooker, E. S. Garlid, J. Zhang, K. S. M. Reddy, S. D. Flexner, C. J. Palmstrom, and P. A. Crowell, Nature Physics **3**, 197 (2007).

[9] F. Nasirpouri and A. Nogaret, *Nanomagnetism and Spintronics: Fabrication, Materials, Characterization and Application* (World Scientific, Singapore, 2011).

[10] Y. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Nature **427**, 50 (2004).

[11] D. Awschalom, N. Samarth, Physics **2**, 50 (2009).

[12] T. Dietl, D. D. Awschalom, M. Kaminska, H. Ohno, *Spintronics* (Academic Press, 2009).

[13] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 2001).

[14] C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Atom-Photon Interactions: Basic Processes and Applications* (Wiley, Weinheim, 2004).

[15] O. V. Kibis, EPL **107**, 57003 (2014).

[16] S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. Shelykh, Phys. Rev. B **91**, 155312 (2015).

[17] M. W. Wu, J. H. Jiang, M. Q. Weng, Phys. Rep. **493**, 61 (2010).

[18] M. M. Glazov, E. Ya. Sherman, V. K. Dugaev, Physica E **42**, 2157 (2010).

[19] W. J. H. Leyland, R. T. Harley, M. Henini, A. J. Shields, I. Farrer, D. A. Ritchie, Phys. Rev. B **76**, 195305 (2007).

[20] C. Tahan and R. Joynt, Phys. Rev. B **71**, 075315 (2005).

[21] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976).

[22] W. A. Harrison, *Solid State Theory* (McGraw-Hill, New York, 1970).

[23] I. S. Gradstein and I. H. Ryzhik, *Table of Series, Products and Integrals* (Academic Press, New York, 2007).

[24] L. D. Landau, E. M. Lifshitz, *Quantum Mechanics:Non-Relativistic Theory* (Pergamon Press, Oxford, 1991).

[25] A. A. Burkov, A. S. Nunez, and A. H. MacDonald, Phys. Rev. B **70**, 155308 (2004).

[26] G. D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 1981).

[27] A. A. Burkov and D. G. Hawthorn, Phys. Rev. Lett. **105**, 066802 (2010).

[28] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).

[29] T. D. Stanescu and V. Galitski, Phys. Rev. B **75**, 125307 (2007).

[30] E. Akkermans and G. Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambridge University Press, New York, 2007).

[31] G. Gumbs and D. Huang, *Properties of Interacting Low-Dimensional Systems* (Wiley-VCH, Singapore, 2011).

[32] I. V. Lerner, B. L. Althsuler, V. I. Fal'ko, and T. Giamarchi, *Strongly Correlated Fermions and Bosons in Low-Dimensional Disordered Systems* (Kluwer, Dordrecht, 2002).

[33] N. S. Averkiev, L. E. Golub, and M. Willander, J. Phys.: Condens. Matter **14**, R271 (2002).

[34] N. S. Averkiev and L. E. Golub, Semicond. Sci. Technol. **23**, 114002 (2008).

[35] P. Boross, B. Dora, A. Kiss, and F. Simon, Sci. Rep. **3**, 3233 (2013).

[36] R. J. Elliott, Phys. Rev. **96**, 266 (1954).

[37] Y. Yafet, in *Solid State Physics, Vol. 14* (Academic Press, New York, 1963).

[38] M. I. D'yakonov and V. I. Perel', Sov. Phys. JETP **33**, 1053 (1971).

[39] T. Ando, A. B. Fowler, and F, Stern, Rev. Mod. Phys. **5**4, 437 (1982).